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**SIMPOZIJUM  
O OPERACIONIM ISTRAŽIVANJIMA  
SYMPOSIUM  
ON OPERATIONS RESEARCH**

**Urednik / Editor  
Slobodan Vujić**





# XXXI SIMPOZIJUM O OPERACIONIM ISTRAŽIVANJIMA XXXI SYMPOSIUM ON OPERATIONS RESEARCH

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The formation of the discrete mathematical model as the precondition of the ore homogenization by opal breccia for the cement industry demands is based on the principle of the located part of the extant. By means of that

## Introduction

**Abstract:** The formation of the discrete mathematical model or the finite differences - modified model as the precondition of the ore homogenization by opal breccia for the cement industry demands are based on the principle of the discrete extant, i.e. the deposit simulation as an extant phenomenon over the mini blocks system. By means of that physical represented model, it may be described mathematically by matrix form symbols or figures which represent the block segment of the open pit. The model formation course of the deposit is run across the few phases.

**KEYWORDS:** DISCREET INTERPOLATION, FINITE DIFFERENCES, MODEL, MINI BLOCK

**Sažetak:** Svaranje modela diskretne interpolacije ili modifikovanog modela konačnih razlika kao preduslov za homogenizaciju rude opalske breče za potrebe cementne industrije, baziraju se na principu postojanja diskrecije tj. simulacije ležišta kao jednog postojećeg fenomena preko sistema mini blokova. Pomoću ovog fizički reprezentativnog modela, može se matematički opisati matičnom formom simbola ili figura, koji predstavljaju blok segmente površinskog kopa. Kurs svaranja modela ležišta se izvodi kroz nekoliko faza.

**KLJUČNE REČI:** DISKRETNA INTERPOLACIJA, KONACNE RAZLIKE, MODEL MINI BLOK

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# The mathematical modelling and computer application for the open pit performance Matematičko modeliranje i primena kompjutera pri izvedbi površinskog kopa



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je uporedbе sa koregistranjem u da se u rudarstvu kada su učinjene, se izražava čak bilo šta da se kove rada odnosno dati moraju imati ve godine unapred, aznanja o tome šta sti, sa aspekta normalija, kvaliteta era je i mogućnost odnosno izrade ma korisnika, sa d i postigne veća stranju podataka i

physical represented model, it may be described mathematically by matrix form symbols or figures whichever representing the block. In such away the extant model directly may be represented as a three-dimensional matrix or indirectly by two-dimensional matrix set in the horizontal or vertical sections.

The method of finite differences is based on shift of partial derivatives with answering differences of relation by answering independent variables.

The method of finite differences for approximate determination of partial differential equations is based on the following:

- Boundary district, in which is looking for determination, is covered with approximate net composed of equal elementary surfaces.
- The partial equation which is given, is shifting in the knots of the net with answering equations in the shape of finite differences.
- On the base of boundary conditions is approving the value of the determinations of boundary knots.
- The system of approximate equations is determining, which present algebraic system with great number of unknowns.
- The determinations of the system of approximate equations is taking as a near determinations of partial differential equations.

### The application of the discrete interpolation

If the purpose is to form the discrete model of both deposit or ore body with the surrounding follower rocks, the deposit extant have to be derived in mini blocks. According to the obtained in formation by the investigated tests have to define mining-geology signs of every block, i.e. the useful component assay, the both assays tailings and injurious components, the digging residence etc. which will contribute for the studying of the possibility of composite material production which will be used in the cement industry. The

idea of the extant discrete interpolation is based on the definition from the influence of every point bearing the investigated information from the influenced group of the investigated mini block. (Fig. 1.)

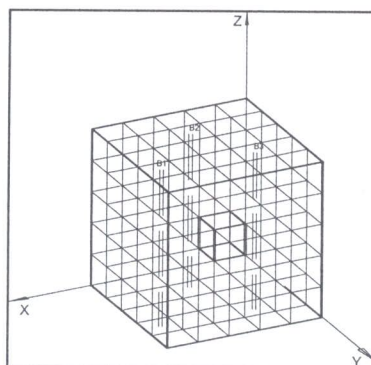


Figure 1. Scheme of the mini block

The influence of the all known points - holes from the influenced group, to the observed unknown point - mini block is estimated according to the equation:

$$U_n = \frac{\sum_{p=1}^{p=m} U_p \cdot L_{p,n}^{-1}}{\sum_{p=1}^{p=m} L_{p,n}^{-1}}$$

where are:

- $U_n$  - unknown point;
- $U_p$  - known point;
- $L_{p,n}$  - distance between influenced known and unknown point;
- $l$  - degree of influenced activity of the distance.

From the theoretical view point, defining the characteristics of every unknown point - the mini block by means of discrete interpolation, the influenced activity has had all information points as a bearers of the mining-geology information about the unique extant whole - deposit, having:

$$U_{\min} < U_n < U_{\max}$$

### The application of the

In physical sense, differences is based on of continue surface approximate presentation system of points arrangement. The idea of surface interpretation by geometric modeling of deposit traits from different nature, as change of contents of useful minerals. With approximate  $U=U(x,y)$  with method which means forming starting of known conditions can be defined with knots) or with boundary lines).

The principle of opportunity for defining parameters for each point. This means that when model of the deposit the base of the mini with the size and surface ( $\Delta x \Delta y$ ), it is net to coincide with points aim) points of If this condition is satisfied  $= U(x_i, y_i)$ , can be block with coordinates

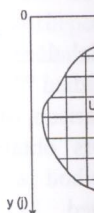


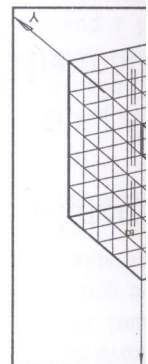
Figure 2. Scheme of covering of t

Laplace's equation for dependent variable boundary district has

$$u_{i,j} = \frac{1}{4} (u_{j,i-1} + u_{j,i+1} + u_{j-1,i} + u_{j+1,i})$$



Interpolation is based on influence of every point of information from investigated mini



own points - holes, to the observed block is estimated

influenced known activity of the

point, defining the known point - the cret interpolation, and all information ne mining-geology ue extant whole -

### The application of the finite differences

In physical sense, the method of finite differences is based on treatment of discrete of continue surface  $U=U(x,y)$ , it means her approximate presentation through approximate system of points arranged about regular net. The idea of surface  $U=U(x,y)$  about interpretation by geologic-mining aspect and modeling of deposits can be connected with nature, as change of strength of layer, change of contents of useful minerals, etc.

With approximate presentation on surface  $U=U(x,y)$  with method of finite differences, which means forming the model of deposit, it is starting of known contour conditions, which can be defined with research holes (boundary knots) or with underground research works (boundary lines).

The principle of discretion is giving opportunity for define the values of searching parameters for each point of the net. This means that when it is forming a discrete model of the deposit, the size and the shape of the base of the mini block needed to coincide with the size and the shape of elementary surface ( $\Delta x \Delta y$ ), it is necessary the knots of the net to coincide with gravity (center of gravity, points aim) points of mini blocks.

If this condition is satisfying, than value for  $U_i = U(x_i, y_i)$ , can be given on answering mini block with coordinates  $(x_i, y_i)$ .

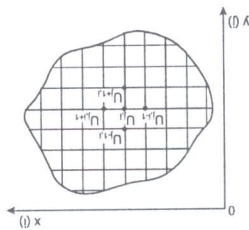


Figure 2. Scheme of geometry interpretation for covering of the boundary district

Laplace's equation for definition of the value of dependent variable  $U=U(x,y)$  in each point of boundary district has the shape:

$$U_{ij} = \frac{1}{4}(U_{j-1,i} + U_{j+1,i} + U_{j,i-1} + U_{j,i+1})$$

### Experimental investigation of discrete interpolation and finite differences

The practical experiments and reviewing of the applicable for the deposit model formation, are carried out by means of experimental investigation from the opal breccia deposit "Spancevo"-Cisimovo-R. Macedonia, according to the set theoretical principles, simultaneously, using the graphic presentation with SURFER computer programme.

The disposition of the investigated holes,  $SiO_2$  and  $Al_2O_3$  assays

Table 1. Chemical analysis results

Hole No.	Samp. No.	Type of material	$SiO_2$	$Al_2O_3$
1	1/1	Opal breccia	74.06	12.81
2	2/1	Opal breccia	79.92	7.54
3	3/1	Opal breccia	87.50	2.15
4	4/1	Opal breccia	85.49	2.92
5	5/1	Opal breccia	80.70	10.06
6	6/1	Opal breccia with and. incl.	53.46	20.13
7	7/1	Opal breccia	79.92	7.54
8	8/1	Opal breccia	84.50	5.15
9	9/1	Opal and tuf breccia	74.95	12.60
10	10/1	Andezite, volcanic tuf	51.80	4.50
11	11/1	Opal breccia with and. incl.	52.70	16.05
12	12/1	Opal breccia with and. incl.	57.10	22.32
13	13/1	Opal breccia	70.36	14.80
14	14/1	Opal breccia	81.70	9.06
15	15/1	Opal breccia	78.92	6.50
16	16/1	Opal breccia	88.40	2.25
17	17/1	Opal breccia	83.17	4.52
18	18/1	Opal breccia	80.94	6.03
19	19/1	Opal breccia with and. incl.	56.40	15.73
20	20/1	Opal breccia	87.30	2.82
21	21/1	Opal and tuf breccia	78.40	6.35
22	22/1	Opal and tuf breccia	73.95	1.60
23	23/1	Opal and tuf breccia	79.70	9.06
24	24/1	Opal breccia	89.40	1.92
25	25/1	Opal breccia	80.93	8.90
26	26/1	Opal and tuf breccia	68.60	10.00
27	27/1	Opal and tuf breccia	69.50	9.10
28	28/1	Opal breccia	86.40	2.25
29	29/1	Opal and tuf breccia	69.60	2.92
30	30/1	Opal breccia	78.82	7.50
31	31/1	Opal and tuf breccia	74.40	6.40
32	32/1	Opal and tuf breccia	73.95	2.65
33	33/1	Opal and tuf breccia	74.82	5.77
34	34/1	Opal breccia	76.60	4.00
35	35/1	Opal and tuf breccia	71.36	13.80
36	36/1	Opal and tuf breccia	78.30	7.25

The opal breccia open mine determination is determined by means of computer programmes for discrete interpolation and finite differences.



It was encircled the deposit investigation field represented by blocks matrix with following dimension:  $Dx = 50$  m and  $Dy = 50$  m with carried in disposition of the investigated holes (the number of 36) with following characteristics (Table 1.)

Matrix obtained from discreet interpolation method ( $SiO_2$ ) Table 2.1.

82.13	68.02	62.15	<b>57.10</b>
<b>84.50</b>	70.55	64.34	58.79
82.50	73.28	68.01	63.88
77.78	75.85	72.36	69.12
75.14	78.60	75.27	71.96
74.74	<b>79.92</b>	76.04	73.61
76.81	78.90	77.05	75.07
79.60	78.33	76.83	74.65
<b>80.70</b>	78.41	76.88	<b>74.06</b>

Matrix obtained from finite differences method ( $SiO_2$ ) Table 2.2.

77.17	68.62	65.33	<b>57.10</b>
<b>84.50</b>	70.47	67.30	63.82
78.18	72.20	69.44	67.34
75.75	73.97	71.47	69.71
74.82	76.19	73.31	71.51
74.82	<b>79.92</b>	74.83	72.91
75.67	78.24	75.75	73.92
77.40	78.14	76.44	74.74
<b>80.70</b>	78.64	77.20	<b>74.06</b>

Matrix obtained from discreet interpolation method ( $SiO_2$ ) Table 3.1.

<b>74.82</b>	77.33	77.93	<b>78.82</b>
75.34	77.92	78.02	78.35
76.33	78.95	78.36	77.66
76.99	80.21	78.31	76.82
77.90	<b>80.93</b>	78.29	75.98
79.54	80.18	77.40	74.98
82.70	78.97	75.99	72.74
87.06	78.17	74.19	70.00
<b>89.40</b>	77.62	73.41	<b>68.60</b>

Matrix obtained from finite differences method ( $SiO_2$ ) Table 3.2.

<b>74.82</b>	76.81	77.20	<b>78.82</b>
75.45	77.22	77.28	77.41
76.07	77.81	77.32	76.72
76.78	78.80	77.34	76.21
77.63	<b>80.93</b>	77.24	75.67
78.76	79.07	76.73	74.96
80.40	78.32	76.06	73.94
83.20	77.93	75.37	72.25
<b>89.40</b>	77.63	74.88	<b>68.60</b>

Matrix obtained from discreet interpolation method ( $SiO_2$ ) Table 4.1.

<b>87.30</b>	78.71	80.68	<b>83.17</b>
85.17	79.42	80.79	82.96
81.43	79.94	80.79	82.13
78.70	80.55	80.67	81.87
76.41	<b>80.94</b>	80.40	81.53
72.92	80.26	80.08	80.96
67.24	78.63	79.84	80.32
59.71	77.44	79.42	79.26
<b>56.40</b>	77.11	79.34	<b>78.92</b>

Matrix obtained from finite differences method ( $SiO_2$ ) Table 4.2.

<b>87.30</b>	78.70	79.44	<b>83.17</b>
82.16	79.31	79.98	81.68
79.72	79.62	80.24	81.35
77.97	79.95	80.32	81.29
76.23	<b>80.94</b>	80.23	81.20
74.12	78.85	79.82	80.94
71.21	77.64	79.35	80.53
66.49	76.94	79.00	79.95
<b>56.40</b>	76.82	78.94	<b>78.92</b>

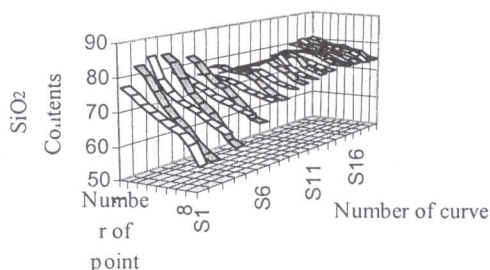


Figure 3. Comparison graphic (table 2.1 & table 2.2)

## Discussion

It's forced the question: "How much the obtained mathematic model does response on the real system which is studied?"

We will make the comparison of the values from the matrix members obtained by the discreet interpolation method and from the matrix members obtained by the finite differences method.

In order to perceive the differences between values obtained by both methods, are separated parts - matrix with sizes  $9 \times 9$ , with fixed points. The biggest part of the obtained results show that there aren't differences and deviation by using of these two methods.

The differences are 1.0% to 1.01%.

The graphic present

## Conclusion

The main aim from of deposit comes d understanding; for On the basis of the reference data rela discreet interpolatio may be seen the fac useful which is first flexibility and acc appropriate in depos change in the mo deposits with ex occurrence, the sto The only issue tha defining of the zone of its shape and siz by intuition whic Therefore, a scient the defining of the been developed. action of the dista component in the dumping action. becomes greater i been defined with l

The exactness of modelling of depo depends from bound



Table 4.1.

68	83.17
79	82.96
79	82.13
67	81.87
40	81.53
08	80.96
84	80.32
42	79.26
34	78.92



Number of curve

44	83.17
98	81.68
24	81.35
32	81.29
23	81.20
82	80.94
35	80.53
00	79.95
94	78.92

Table 4.2.

"How much the does response on rison of the values s obtained by the od and from the differences between thods, are separated, with fixed points. s and deviation by

ion graphic (p. 2.2)

The differences are moved in the limits from 1.0% to 1.01%. The graphic presentation confirm this.

### Conclusion

The main aim from the mathematic modelling of deposit comes down to three basic function: understanding; foreseeing; control.

which in primary phase of modeling on the base of carefully done process of research informations (information points) and the choice of approximate net precisely are defined, and of the number of iterations is done in the determination of equations by Gauss-Seidl's method.

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- The exactness of the results obtained by modelling of deposit in this method, mainly depends from boundary (contour) conditions, been defined with less accuracy.
- The importance of this one component in the model which exerts a action of the distance is essential important been developed. The degree of influential the defining of the zone of influence has not Therefore, a scientific objective procedure for by intuition which may cause mistakes. of its shape and size is made experimentally or defining of the zone of influence. The selection The only issue that remains unsolved is the occurrence, the stockwork deposits etc. deposits with explicit effect of native change in the modelled trait, for example appropriate in deposits which poses an abrupt flexibility and accuracy. The method isn't useful which is first of all seen in its simplicity, may be seen the fact that this method is fairly discrete interpolation in modelling of deposits reference data related to the application of On the basis of the obtained results and other understanding; foreseeing; control.